

# Spin torque antiferromagnetic nanooscillator in the presence of the magnetic noise

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Spin-torque effects in antiferromagnetic (AFM) materials are of great interest because of the possible applications as high-speed spintronic devices. In the present paper we analyze the statistical properties of the current-driven AFM nanooscillator that result from the white Gaussian noise of magnetic nature. Starting from the peculiarities of deterministic dynamics we derive Langevin and Fokker-Planck equations in energy representation for two normal modes. We find the stationary distribution function in the subcritical and overcritical regimes and calculate the current dependence of average energy, energy fluctuation and their ratio (quality factor). It is found that one of the modes (noncritical) shows the Boltzmann statistics with current-dependent effective temperature in all range of current values. The effective temperature of the other (soft) mode critically depends on current in the subcritical region. Distribution function of soft mode follows the Gaussian law above the generation threshold. In the overcritical regime the full average energy and quality factor grow with current value that makes AFM nanooscillators promising candidates as active spintronic components.

ntiferromagnets, spintronics, thermal noise, Langevin equation, Fokker-Planck equation, current-driving spin-pumping 5.50.Ee, 85.75.-d, 05.40.Ca, 05.10.Gg, 72.25.Pn

## I. INTRODUCTION

Nowadays the spin-polarized current is widely used for manipulation of nano-magnetic structures. Corresponding physical mechanism is based on the so-called spin-transfer-torque (STT) effect, which was predicted by Slonczewski and Berger [1, 2]: a spin-polarized current may transfer angular momentum to a free ferromagnetic (FM) layer resulting in a macroscopic torque on the latter's magnetization. In small FM particles (nanopillars or nanocontacts) in which the magnetization can be assumed to be spatially uniform, the STT results in the steady rotation of the magnetization (see, e.g. [3–6]). This effect is interesting for applications: as a tempting alternative to the electronic (*vs* spintronic) diode, as radio-frequency devices used for telecommunications, as timing mechanisms etc.

Recently it was shown [7] (see also [8]) that the STT effect should take place also in antiferromagnetic (AFM) materials and, in analogy with FMs, should give rise to steady rotation of the Néel (or AFM) vector. Current-controlled AFM nanoparticles are promising candidates for spintronic devices because of high working frequencies that fall into 0.1-1 THz range (for comparison, typical frequencies of FM nanooscillators are 1-50 GHz [9, 10]). However, the central problem for practical applications of nanooscillators is the reduction and control of their linewidth and as a consequence, improvement of their quality factor. Thus, understanding of the stochastic processes (such as, e.g. thermal noise) that set condition for linewidth is increasingly important for operation of STT devices.

Thermal noise in FM particles was a matter of study starting from the pioneering work by W. F. Brown [11], where the noise has been described as a stochastic magnetic field acting on the magnetization and corresponding Fokker-Planck equation was derived. Apalkov and Visscher [12] generalized the nonlinear Fokker-Planck equation to the case of Slonczewski STT. Since then noise properties of the FM-based nonlinear oscillator in the presence of spin-polarized current were studied both experimentally and theoretically [13–16].

Investigation of noise in AFM systems is much more complicated problem compared to that in FMs due to *i*) the larger number degrees of freedom; *ii*) Newtonian-like (*vs* precession-like in FMs) dynamics of the Néel vector. In some cases, e.g. the description of superparamagnetism or switching processes in AFM nanoparticles, the problem can be, in principle, reduced to the description of fluctuations of weak FM moment which inevitably arises due to the surface effects or Dzyaloshinskii-Moriya interactions [17–20].

However, those peculiarities of AFM dynamics that are related with the exchange coupling between the magnetic sublattices, like magnetoelastic effects [21], spin-wave spectra [22], STT phenomena [7] etc, should be described with due account of the variables inherent to AFM ordering. Corresponding approach based on the dynamics of the Néel vector was recently developed [23] for the case of a collinear compensated AFM with two magnetic sublattices. In the present paper we apply this approach to the analysis of the efficiency of AFM nanooscillator conducted by spin-polarized current. We assume that the thermal noise in AFM particle arises from fluctuations of the random magnetic field. Following the method of slow and fast variables [24] and energy representation for non-equilibrium steady state [25] we formulate the Fokker-Planck equation for energy distribution and study the linewidth of spin-torque AFM nanooscillator depending on temperature and current.

## II. MAGNETIC DYNAMICS OF ANTIFERROMAGNETIC PARTICLE IN THE PRESENCE OF SPIN-POLARIZED CURRENT

The system under consideration (see figure 1 a) is an AFM nanoparticle with the elliptic cross-section. The size of the particle is large enough to ensure the AFM ordering and is small enough (below the domain wall thickness, so-called macrospin approximation) to neglect the space variation of the magnetic properties. Hard FM (polarizer) and thin nonmagnetic layers shown in figure 1a) are the ancillary elements that deliver the spin-polarized current to AFM. We assume that the temperature  $T$  of the system is kept constant thus neglecting the thermal (Joule) heating of the system.

For the sake of simplicity we consider a collinear AFM with two equivalent magnetic sublattices  $\mathbf{M}_1$  and  $\mathbf{M}_2$  and disregard weak FM moment that can arise either from the intrinsic properties of material or due to the surface effects. The coupling between the sublattices (characterized with the spin-flip field  $H_E$ ) is assumed to be much stronger than other fields (including anisotropy and all the external fields). To this end, the magnetic state of such an AFM nanoparticle is unanimously described by the only one, AFM vector,  $\mathbf{L} \equiv \mathbf{M}_1 - \mathbf{M}_2$  of the fixed length  $|\mathbf{L}| = 2M_0$ . Corresponding dynamic equations for  $\mathbf{L}$  can be obtained within the standard Lagrange technique, the Lagrange function being [26]

$$\mathcal{L}_{AFM} = \frac{m_L}{2} \dot{\mathbf{L}}^2 + \gamma m_L \left[ \dot{\mathbf{L}} \cdot (\mathbf{L} \times \mathbf{H}) \right] - w_{an}(\mathbf{L}) + \frac{\gamma^2 m_L}{2} (\mathbf{L} \times \mathbf{H})^2, \quad (1)$$

where  $\mathbf{H}$  is an external magnetic field and  $w_{an}(\mathbf{L})$  is the energy of magnetic anisotropy that forms a potential well for AFM vector,  $\gamma$  is the gyromagnetic ratio. The value  $m_L \equiv 1/(2\gamma^2 M_0 H_E)$  plays a role of the “inertia mass” of the

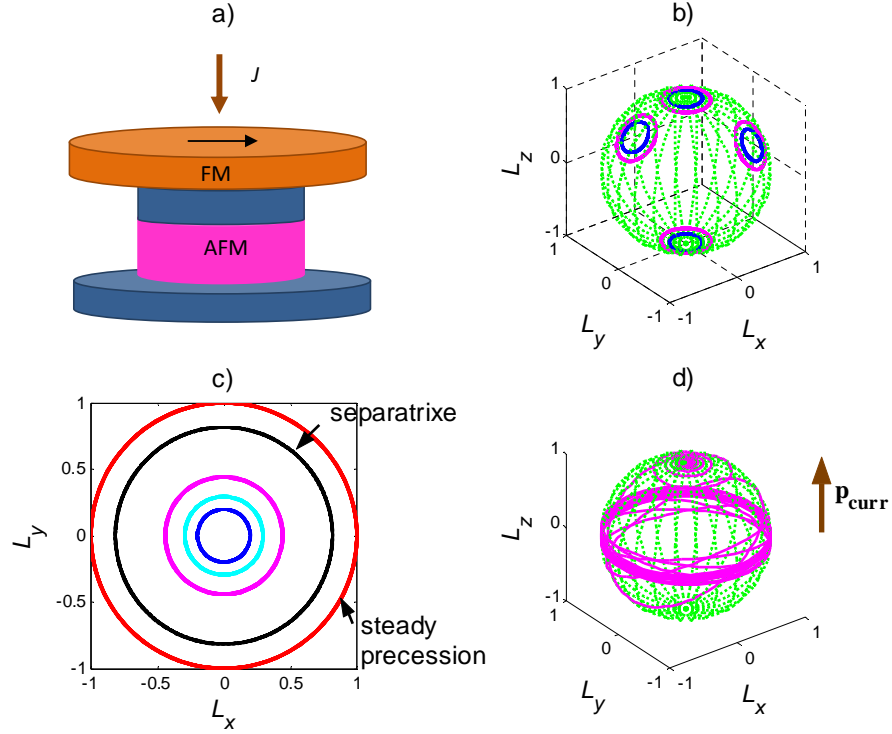


Figure 1. Dynamics of AFM vector in the presence of spin-polarized current. a) AFM particle of the elliptic cross-section is placed between two electrodes, the top electrode being FM (polarizer). The spacer between FM and AFM is nonmagnetic to avoid direct exchange coupling between the magnetic layers. b) Typical trajectories of  $\mathbf{L}$  vector for circularly polarized modes. Different areas correspond to different equilibrium orientations (4 of six possible are shown). c) Projection of trajectories in  $\mathbf{L}$  space to the  $xy$  plane. Three inner circles correspond to the normal modes with different amplitude in the vicinity of equilibrium state  $\mathbf{L} \parallel z$ . The last but one circle corresponds to separatrix and the outer circle is a steady state trajectory. d) Trajectory in the overcritical regime ( $J = 1.2J_{\text{cr}}$ ). In the initial state  $\mathbf{L} \parallel \mathbf{p}_{\text{curr}} \parallel z$ .

Newtonian “material point” with “radius-vector”  $\mathbf{L}$ .

For the definiteness, in what follows we assume that the AFM layer has a cubic magnetic anisotropy modeled with the following expression

$$w_{\text{an}} = -\frac{H_{\text{an}}}{8M_0^3}(L_x^4 + L_y^4 + L_z^4), \quad (2)$$

where the orthogonal axes  $x$ ,  $y$  and  $z$  coincide with the easy directions for the Néel vector,  $H_{\text{an}} \ll H_E$  is anisotropy fields.

Dissipation is modeled with the Rayleigh function which in the presence of spin-polarized current  $J$  takes a form [7]:

$$\mathcal{R}_{\text{AFM}} = \gamma_{\text{AFM}} m_L \dot{\mathbf{L}}^2 - \frac{\sigma J}{2\gamma M_0} [\mathbf{p}_{\text{curr}} \cdot (\mathbf{L} \times \dot{\mathbf{L}})]. \quad (3)$$

Here the first term models the internal damping, damping coefficient  $2\gamma_{\text{AFM}}$  is, in fact, the AFMR linewidth, the constant  $\sigma = \hbar\gamma\varepsilon/(2eM_0v_{\text{AFM}})$  is proportional to the efficiency  $\varepsilon$  of the spin transfer processes,  $v_{\text{AFM}}$  is the volume of AFM nanoparticle,  $\hbar$  is the Plank constant,  $e$  is the electron charge. Unit vector  $\mathbf{p}_{\text{curr}}$  is parallel to the direction of the current spin polarization.

Deterministic behaviour of such a system in the presence of current  $J$  polarized along one of the easy axes,  $\mathbf{p}_{\text{curr}} \parallel z$ , was analyzed in details in reference [7]. Some characteristic features important for further consideration are summarized in figure 1(b-d). In the so-called subcritical regime,  $|J| < J_{\text{crit}} \equiv 2\gamma_{\text{AFM}}\Omega_{\text{AFMR}}/(\gamma\sigma H_E)$  (where  $\Omega_{\text{AFMR}} \equiv \gamma\sqrt{H_E H_{\text{an}}}$  is AFMR frequency) the AFM vector  $\mathbf{L}$  has three equilibrium orientations close to  $x$ ,  $y$  and  $z$  directions that define six (corresponding to  $\mathbf{L}$  and  $-\mathbf{L}$ ) basins of finite motion in the phase space. Small deflection from the easy axis in equilibrium is due to the current-induced torque. Typical phase trajectories in the vicinity of equilibrium points in nondissipative approximation correspond to the clockwise/counterclockwise rotations of AFM

vector (circles in figure 1b) and could be associated with the circular polarized normal modes with the frequencies  $\pm\Omega_{\text{AFMR}}$ . Spin-polarized current acts as a negative damping for one of the modes (“soft” mode) and as a positive damping for the other. However, below the critical current the negative damping is suppressed by the internal losses and the real phase trajectories are the twisted spirals.

In the overcritical regime ( $|J| > J_{\text{crit}}$ ) the stable (non-equilibrium) state corresponds to steady rotation of AFM vector in  $xy$  plane (perpendicular to  $\mathbf{p}_{\text{curr}}$ ) with the current-dependent frequency  $\omega = (J/J_{\text{crit}})\Omega_{\text{AFMR}}$  (the outer circle in figure 1c). The direction of the rotation coincides with the direction of the soft mode. In the steady state the internal losses are exactly compensated with the current-induced energy pumping. This regime can be associated with power generation.

Trajectory of AFM vector presented in figure 1d) shows the typical current-induced transition from the equilibrium state  $\mathbf{L}\parallel z$  to the steady precession in  $xy$  plane. The prominent feature of this motion is the presence of two well separated time scales: fast rotation around  $z$  axis with the frequency  $\propto \Omega_{\text{AFMR}}$  and slow, with the characteristic time  $\propto 1/\gamma_{\text{AFM}}$  variation of the polar angle from 0 ( $\mathbf{L}\parallel z$ ) to  $\pi/2$  ( $\mathbf{L} \perp z$ ). Deviation from this scenario takes place only in the close vicinity of the separatrix (see figure 1c, next to last circle) where the rotation frequency substantially diminishes.

Thus, in the vicinity of equilibrium and stationary steady states the current-induced dynamics of AFM is characterized with the set of slow and fast variables that allows substantial simplification of the description.

### III. LANGEVIN DYNAMICS AND FOKKER-PLANCK EQUATIONS IN ENERGY REPRESENTATION

While the state of FM nanoparticle is described with only two variables in configuration space, the dynamics of AFM nanoparticle needs for description at least four (with account of normalization condition  $|\mathbf{M}_1| = |\mathbf{M}_2| = M_0$  far below the Néel temperature) variables in the phase space, namely, generalized coordinates  $\mathbf{L}$  and corresponding generalized momenta  $\mathbf{P}_L$ . This results in a rather complicated Langevin equations in the phase space [23]:

$$\begin{aligned}\dot{\mathbf{L}} &= \mathbf{P}_L/m_L - \gamma\mathbf{L} \times \mathbf{h}, \\ \dot{\mathbf{P}}_L &= \mathbf{F}_L + \mathbf{F}_{\text{diss}} - \gamma(\mathbf{P}_L - 2\gamma_{\text{AFM}}m_L\mathbf{L}) \times \mathbf{h},\end{aligned}\tag{4}$$

where  $\mathbf{F}_L \equiv -\partial w_{\text{an}}(\mathbf{L})/\partial\mathbf{L}$  is the potential (gradient) force, and the dissipative force  $\mathbf{F}_{\text{diss}}$  is given by the following expression

$$\mathbf{F}_{\text{diss}} \equiv -\left.\frac{\partial\mathcal{R}_{\text{AFM}}}{\partial\dot{\mathbf{L}}}\right|_{\dot{\mathbf{L}}\rightarrow\mathbf{P}_L} = -2\gamma_{\text{AFM}}\mathbf{P}_L - \frac{\sigma J}{2\gamma M_0}\mathbf{p}_{\text{curr}} \times \mathbf{L}.\tag{5}$$

The thermal noise in equation (4) is modeled with the random magnetic field  $\mathbf{h}(t)$  which we assume to be the white Gaussian noise with

$$\langle\mathbf{h}(t)\rangle = 0, \quad \langle h_j(t_1)h_k(t_2)\rangle = 2D\delta_{jk}\delta(t_1 - t_2),\tag{6}$$

where  $2D$  represents the intensity of thermal fluctuations.

However, the number of the effective phase variables in Langevin, (4), and corresponding Fokker-Planck equations can be reduced if we take into account the above mentioned peculiarities of the dynamics in the presence of spin-polarized current.

First, in the vicinity of equilibrium and stationary states the motion of AFM vector is finite and can be decomposed to a linear combination of two independent (normal) modes (see figure 2). This means that one can use the set of the canonically conjugated variables action-angle,  $I_{\pm}, \varphi_{\pm}$  ( $\pm$  correspond to the clock/counterclockwise rotations in configuration space) instead of coordinates and momenta. Second, in a low-temperature approximation, when the temperature  $T$  is much less than the energy barrier between the different equilibrium/stationary states, all the essential phase trajectories for each mode have the same rotation/oscillation frequency,  $\omega_{\pm}$ . Thus, instead of action one can use the energy of the mode,  $E_{\pm} = I_{\pm}\omega_{\pm}$ , as a canonical variable. At last, two time scales allow one to exclude the angle variables  $\varphi_{\pm}$  by averaging over the period of rotations.

As a result, Langevin equations (4) could be rewritten in terms of the averaged energies  $E_{\pm}$  as follows:

$$\frac{dE_{\pm}}{dt} = -\overline{\mathbf{P}_L \cdot \mathbf{F}_{\text{diss}}} + \gamma \left[ \left( 2\gamma_{\text{AFM}}\mathbf{P}_L - \frac{\partial w_{\text{an}}}{\partial\mathbf{L}} \right) \cdot \mathbf{L} \times \mathbf{h} \right].\tag{7}$$

where the overline means averaging over the period of rotation and the summands in r.h.s. of equation (7) should be expressed in terms of  $E_{\pm}$ . Here the averaged energy is

$$E_{\pm} = \frac{\omega_{\pm}}{2\pi} \int_0^{2\pi/\omega_{\pm}} [\mathbf{P}_L^2/(2m_L) + w_{\text{an}}(\mathbf{L})] dt.\tag{8}$$

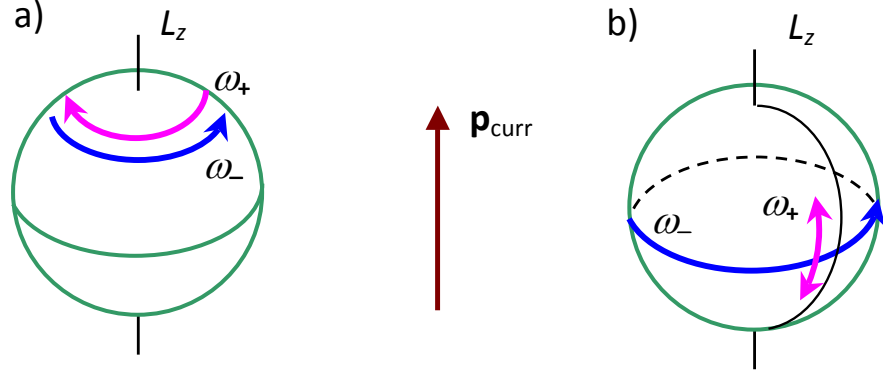


Figure 2. Normal circularly polarized modes in the presence of spin-polarized current in a) subcritical and b) overcritical regimes. Typical trajectories lay over the sphere  $|\mathbf{L}| = 2M_0$  in configuration space.

The explicit closed form of equation (8) will be obtained in next subsections for the limiting cases of subcritical and overcritical regimes.

### A. Subcritical regime, $J < J_{\text{cr}}$

Let us consider the basin of states in the vicinity of equilibrium point  $\mathbf{L} \parallel \mathbf{p}_{\text{curr}} \parallel z$  and chose  $L_x, L_y \ll 2M_0$  as generalized coordinates,  $L_z \approx 2M_0$ . For the normal modes the time dependence of the dynamic variables is given by the expressions

$$L_x + iL_y = 2M_0 c_{\pm} e^{i\omega_{\pm} t}, \quad P_{Lx} + iP_{Ly} = 2iM_0 m_L \omega_{\pm} c_{\pm} e^{i\omega_{\pm} t}, \quad \omega_{\pm} = \pm \Omega_{\text{AFMR}}. \quad (9)$$

The averaged energy is related with the amplitude  $c_{\pm}$  as follows:  $E_{\pm} = 2E_0 c_{\pm}^2$ , where the value  $E_0 \equiv 2M_0^2 \Omega_{\text{AFMR}}^2 m_L = M_0 H_{\text{an}}$  defines the characteristic energy scale for AFM nanoparticle.

Substituting expressions (9) into (5) and (7) we arrive to the system of two independent Langevin equations:

$$\begin{aligned} \frac{d\varepsilon_{\pm}}{dt} = & -2\gamma_{\text{AFM}} \left( 1 \pm \frac{J}{J_{\text{crit}}} \right) \varepsilon_{\pm} + 2\gamma\sqrt{\varepsilon_{\pm}} (\pm h_x \sin \Omega_{\text{AFMR}} t - h_y \cos \Omega_{\text{AFMR}} t) \\ & + \gamma \frac{2\gamma_{\text{AFM}}}{\Omega_{\text{AFMR}}} [2\sqrt{\varepsilon_{\pm}} (h_x \cos \Omega_{\text{AFMR}} t \pm h_y \sin \Omega_{\text{AFMR}} t) \mp \varepsilon_{\pm} h_z], \end{aligned} \quad (10)$$

where  $\varepsilon_{\pm} \equiv E_{\pm}/E_0$  is a dimensionless energy.

As it is seen from equation (10), the clock/counterclockwise modes (figure 2a) interact with the current in different ways. If  $J > 0$ , the effective damping of the first mode (with the energy  $E_+$ ) increases and that of the second (with the energy  $E_-$ ) decreases, due to the action of spin-polarized current.

Equations (10) are the typical Langevin equations in energy representation considered in details in [25]. The first summand in the r.h.s. describes the rate of the direct energy exchange which depends upon the current value  $J$ . All but first summands in the r.h.s. of (10) account for system-environment interaction induced by field fluctuations. The diffusion functions (coefficients before  $h_j$ ) depend on energy and thus correspond to multiplicative noise. Last two terms (in square brackets) are multiplied by small factor  $\gamma_{\text{AFM}}/\Omega_{\text{AFMR}} \ll 1$  and will be neglected for the sake of simplicity.

In the accepted approximation of noninteracting modes the distribution function in phase space,  $f(\mathbf{L}, \mathbf{P}_L; t)$ , can be factorized as follows:  $f(\mathbf{L}, \mathbf{P}_L; t) \Rightarrow f_+(E_+; t)f_-(E_-; t)$ . The Fokker-Planck equations for  $f_{\pm}$  are then deduced in a standard manner from (10) in Stratonovich convention and take a form:

$$\frac{\partial f_{\pm}(E_{\pm})}{\partial t} = \frac{\partial}{\partial E_{\pm}} \left\{ \left[ \gamma^2 D E_0 \sqrt{E_{\pm}} \frac{\partial}{\partial E_{\pm}} \sqrt{E_{\pm}} + 2\gamma_{\text{AFM}} \left( 1 \pm \frac{J}{J_{\text{crit}}} \right) E_{\pm} \right] f_{\pm}(E_{\pm}) \right\}. \quad (11)$$

From the stationary solution of (11) one gets the AFM probability distribution function  $f(E_+, E_-) = f_+(E_+)f_-(E_-)\sqrt{E_+E_-}$  (with due account of Jacobian):

$$f(E_+, E_-) = f_0 \exp \left\{ -\frac{2\gamma_{\text{AFM}}}{\gamma^2 D E_0} \left[ \left( 1 + \frac{J}{J_{\text{crit}}} \right) E_+ + \left( 1 - \frac{J}{J_{\text{crit}}} \right) E_- \right] \right\}, \quad (12)$$

where  $f_0$  is a normalization constant and the diffusion coefficient is related with the temperature  $T$  through the fluctuation-dissipation theorem as follows:

$$D = \frac{2\gamma_{\text{AFM}}}{\gamma^2 E_0} T. \quad (13)$$

### B. Overcritical regime, $J > J_{\text{cr}}$

In the overcritical regime the motion can be decomposed (like it was done in the reference [27] for FM) into steady rotation of AFM vector in  $xy$  plane with the frequency  $\omega_-$  and small meridian oscillations of  $\mathbf{L}$  vector with the frequency  $\omega_+$  (see figure 2b).

The “+”-mode is parametrized as follows

$$\begin{aligned} L_z &= c e^{i\omega_+ t}, \quad P_{Lz} = 2iM_0 m_L \omega_+ c e^{i\omega_+ t}, \\ \omega_+ &= \Omega_{\text{AFMR}} \sqrt{\frac{3}{4} + \frac{J^2}{J_{\text{cr}}^2}}, \quad E_+ = 2E_0 \left( \frac{3}{4} + \frac{J^2}{J_{\text{cr}}^2} \right) c^2. \end{aligned} \quad (14)$$

Corresponding Langevin and Fokker-Planck equations are analogous to (10) and (11).

Parametrization of “-”-mode coincides with that in equation (9) with  $c_- = 1$ . In this particular case the proper dynamic variable is action  $I_- = 8\pi M_0^2 m_L \omega_-$ . Corresponding Langevin equation takes a form

$$\frac{dI_-}{dt} = -2\gamma_{\text{AFM}}(I_- - 8\pi M_0^2 m_L \omega_-^{(0)}) - \gamma I_- h_z, \quad (15)$$

where  $\omega_-^{(0)} = -(J/J_{\text{cr}})\Omega_{\text{AFMR}}$  is the deterministic frequency of steady rotation. Transition to energy representation can be obtained directly from (15) by substitution  $I_- = 4\pi M_0 \sqrt{2m_L E_-}$ :

$$\frac{dE_-}{dt} = -4\gamma_{\text{AFM}} \left( E_- - \frac{J}{J_{\text{cr}}} \sqrt{E_0 E_-} \right) - 2\gamma E_- h_z. \quad (16)$$

Corresponding Fokker-Planck equation is then deduced as following:

$$\frac{\partial f_-(E_-)}{\partial t} = \frac{\partial}{\partial E_-} \left\{ \left[ 4\gamma^2 D E_- \frac{\partial}{\partial E_-} E_- + 4\gamma_{\text{AFM}} \left( E_- - \frac{J}{J_{\text{cr}}} \sqrt{E_0 E_-} \right) \right] f_-(E_-) \right\}. \quad (17)$$

Ultimately, the stationary AFM distribution function in the overcritical regime takes a form:

$$f(E_+, E_-) = f_0 \exp \left[ -\frac{4E_+}{[3 + 4(J/J_{\text{crit}})^2]T} - \frac{(E_- - E_0 J^2/J_{\text{cr}}^2)^2}{2TE_0 J^2/J_{\text{cr}}^2} \right], \quad (18)$$

where we have taken into account the relation (13) and assumed that  $T \ll E_0$ .

## IV. DISCUSSION

In the previous section we derived the stochastic equations for the current-controlled AFM nanoparticle in the low temperature approximation. The energy scale of the system is given by the magnetic anisotropy energy  $E_0$ , so, the validity of equations obtained is limited by inequality  $T \ll E_0$ . Taking into account that characteristic value of magnetic anisotropy for the AFMs with high Néel temperature is  $10^3 \div 10^4$  J/m<sup>3</sup> (see, e.g. [28] for Mn<sub>82</sub>Ni<sub>18</sub>) and typical size of nanoparticle is  $50 \times 50 \times 5$  nm<sup>3</sup> one gets the following estimation:  $E_0 \propto 10^{-20} \div 10^{-19}$  J. Thus, the proposed model can be applied up to the room temperature,  $T_{\text{RT}} = 4 \cdot 10^{-21}$  J.

We found the stationary solutions of Fokker-Planck equations that describe the distribution function in the vicinity of equilibrium, (12), and nonequilibrium steady, (18), states where the swapping between these two states can be neglected.

For the noncritical (“+”) mode the probability function  $f(E_+)$  (see (12) and (18)) follows the Boltzmann law with the current-dependent effective temperature

$$T_{\text{eff}}^{(+)} = T \begin{cases} (1 + J/J_{\text{crit}})^{-1}, & J < J_{\text{crit}}, \\ 0.75 + J^2/J_{\text{crit}}^2, & J > J_{\text{crit}}. \end{cases} \quad (19)$$

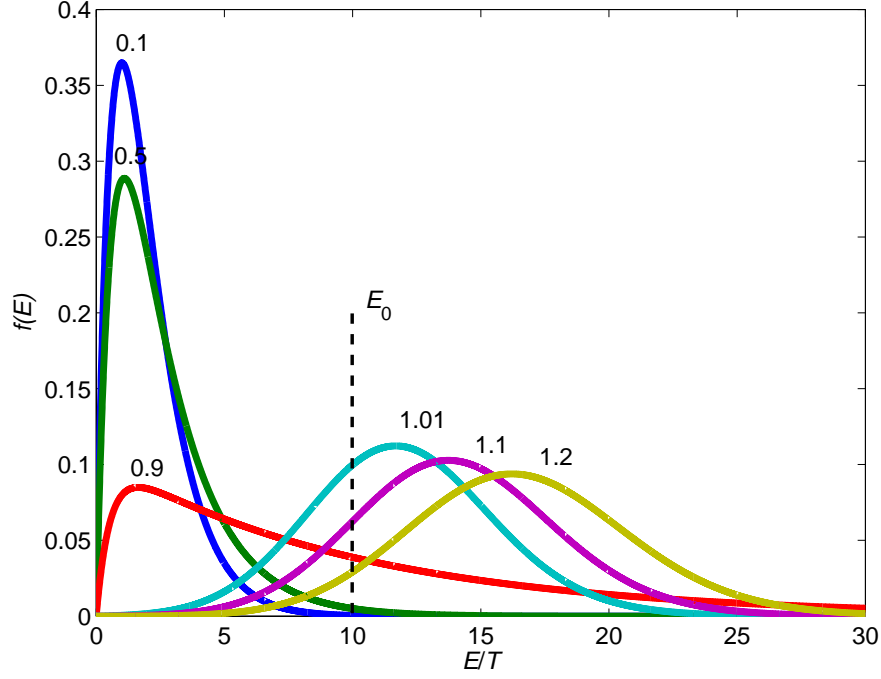


Figure 3. Energy distribution function for different current values  $J/J_{\text{cr}}$  (shown with numbers near the curves) in the subcritical ( $J < J_{\text{cr}}$ ) and overcritical ( $J > J_{\text{cr}}$ ) regimes. Energy is measured in the dimensionless units,  $E/T$ . Vertical dashed line shows the energy  $E_0 = 10T$  in the overcritical regime.

The “soft” (“-”) mode shows the Boltzmann-like distribution in the subcritical regime ( $J < J_{\text{crit}}$ ) which changes to the Gaussian-like with the current-dependent average,  $E^{-(0)} \equiv E_0 J^2 / J_{\text{crit}}^2$ , and current-dependent dispersion,  $TE_0 J^2 / J_{\text{crit}}^2$  at  $J > J_{\text{crit}}$ . In the subcritical region the effective temperature  $T_{\text{eff}}^{(-)} = T / (1 - J/J_{\text{crit}})$  diverges as  $J \rightarrow J_{\text{crit}} - 0$ . This singularity can be avoided with due account of the swap processes (see trajectory in figure 1 d) that are important in the vicinity of critical current  $J \approx J_{\text{crit}}$ . However, this problem is out of scope of this paper.

It should be mentioned that qualitatively the statistical properties of the “soft” mode in AFM particle are analogous to those of FM nanoparticle in the presence of spin-polarized current [13]: Boltzmann distribution and critical behaviour of the effective temperature in subcritical region, and Gaussian-like distribution in overcritical regime. However, even for one AFM mode the current dependencies of average energy and dispersion are different, as will be discussed below.

From the practical point of view more informative is distribution in full energy,  $E = E_+ + E_-$ , of AFM nanoparticle. Corresponding distribution function,  $f(E)$ , calculated from (12) and (18) as a conventional probability, is shown in figure 3.

Analysis of the curves in figure 3 calculated for different current values shows that in the subcritical regime ( $J/J_{\text{cr}} = 0.1, 0.5, 0.9$ ) the distribution function is asymmetric with maximum at

$$E_{\text{max}} = \frac{T}{J/J_{\text{crit}}} \tanh^{-1} \frac{J}{J_{\text{crit}}} \approx T. \quad (20)$$

Average energy of the nanoparticle,  $E_{\text{av}} \equiv \langle E \rangle = T_{\text{eff}}^{(+)} + T_{\text{eff}}^{(-)}$ , consists of the noisy component only and diverges as  $J \rightarrow J_{\text{crit}}$ . Energy fluctuation,

$$\Delta E \equiv \sqrt{\langle (E - E_{\text{av}})^2 \rangle} = \left( T_{\text{eff}}^{(+)} + T_{\text{eff}}^{(-)} \right) \sqrt{\frac{1}{2} \left( 1 + \frac{J^2}{J_{\text{crit}}^2} \right)} \quad (21)$$

diverges in the same way (see figure 4 a). Thus, the quality factor,  $Q = E_{\text{av}} / \Delta E$  diminishes down to 1 as  $J \rightarrow J_{\text{crit}} - 0$ . This tendency is quite obvious if one takes into account the noisy source of the energy in the system. On the other hand, the presence of singularity in  $\Delta E$  shows that thermal noise can play an important role in the transition from equilibrium to nonequilibrium steady state in the vicinity of critical current.

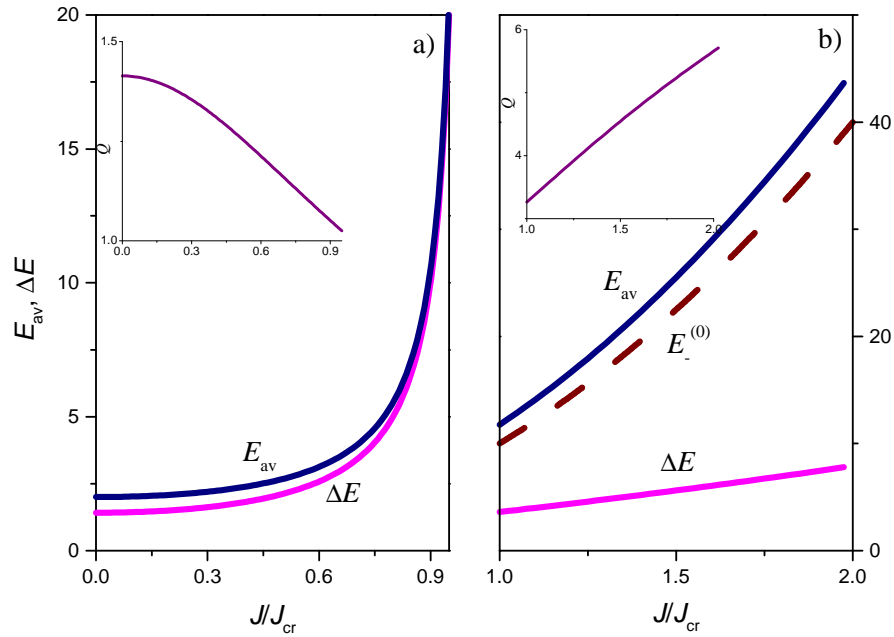


Figure 4. Current dependence of the average energy,  $E_{av}$ , and energy fluctuations,  $\Delta E$  in a) subcritical ( $J < J_{cr}$ ) and b) overcritical ( $J > J_{cr}$ ) regimes. Insets show the current dependence of the quality factor,  $Q = E_{av}/\Delta E$ . The energy  $E_0 = 10T$ . Dashed line shows the current-dependence of the “soft” mode energy  $E_-^{(0)} = E_0 J^2 / J_{cr}^2$ .

In the overcritical regime the distribution function  $f(E)$  has a Gaussian-like shape with the maximum close to the average energy  $E_{av}$ . Both  $E_{av}$  and  $\Delta E$  grow with current (see figure 4 b). However, the main contribution into  $E_{av}$  arises from the deterministic (low entropy) current-induced rotation of AFM vector, while  $\Delta E$  originated from the noise slightly intensified by the current. Thus, in this case the quality factor  $Q$  is finite at  $J = J_{cr}$  and increases with current almost linearly (inset in figure 4 b). This behaviour crucially differs from that for FM nanoparticle. Namely, in supercritical regime quality factor of FM oscillator,  $Q_{FM} \propto \sqrt{1 - J_{cr}/J}$ , vanishes in the close vicinity of  $J_{cr}$ . This opens a way for the potential applications of AFM nanoparticles as active elements of spintronics devices and as a possible alternative to FM nanooscillators. However, further development of the model is necessary to account for Joule heating, current fluctuations etc.

In summary, we considered the current-induced dynamics of AFM nanoparticle in the presence of white Gaussian noise which originates from the random magnetic fields. We found the stationary energy distribution functions in two regimes: subcritical, when the spin-polarized current is too small to reorient AFM vector from the initial equilibrium state, and overcritical, when the spin-polarized current keeps steady rotation of AFM vector. Average energy and energy fluctuations in the subcritical regime show the critical behaviour as  $J \rightarrow J_{crit}$ . This can be used to facilitate the current-induced reorientation of AFM vector. In the overcritical regime the quality factor of AFM particle as nanooscillator can be increased by adjusting the current value.

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